**Non-linear system of simultaneous equations**

**1.** Solve for real numbers :

From ,

The solution is

**2.** Solve for real numbers:

By inspection, is a solution of first and fourth equations. By substitution, it also satisfies the second and third equations. Since the equations are symmetric, all permutations are solutions, that is

However, these are all the solutions since the product of the degrees of the equations is

**3.** Given:

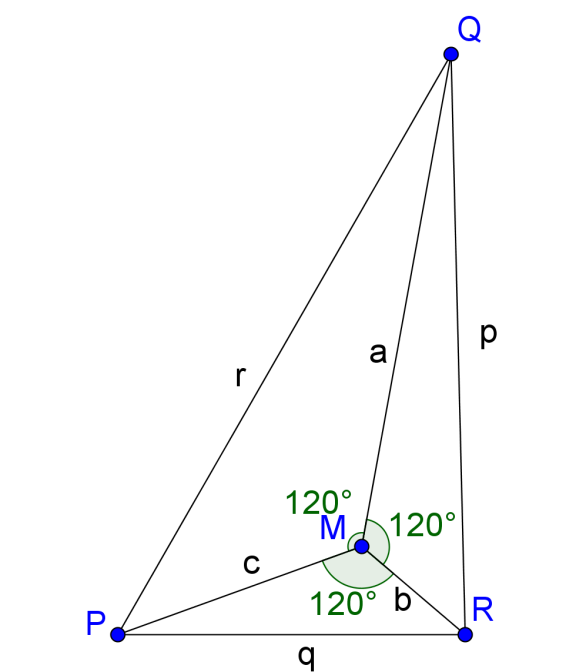
**(a)** If , find .

**(b)** **(Hard)** If are real numbers, find .

(a) Put

Since

By the converse of Pythagoras Theorem, we can form right-angled at .

 is a point inside such that

**(b) (i)** If , let then the equation becomes:

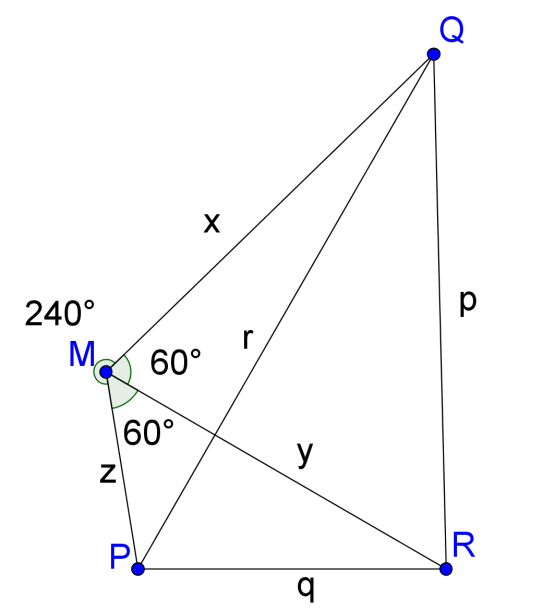
which is similar to the original set of equation,

**(ii)** If , let then the equation becomes:

Put

Since

By the converse of Pythagoras Theorem, we can form right-angled at .

 is a point outside such that

**(iii)** If , let

we can use similar method as in **(ii)**. (Readers may try.)

and can get

**(iv)** We don’t have solutions for other cases such as .

**Tough readers may also try to find the values for a, b, c.**

**I include here the complete solution.**

a~~-1.00909,   b~~3.37444,   c~~-4.4185,   d~~-13.8564

a~~1.00909,   b~~-3.37444,   c~~4.4185,   d~~-13.8564

a~~-2.354,   b~~-1.02391,   c~~-3.38852,   d~~13.8564

a~~2.354,   b~~1.02391,   c~~3.38852,   d~~13.8564

**4. (Hard)** Solve for real numbers :

The system is cyclic, without loss of generality, let .

If , then

, ( may be 0)

Therefore

Putting in the original system confirms the result.

Since the system is cyclic, if , we can still give as solution.

Thus we can assume distinct values of , that is, .

Then we can replace

So if (Note that , this holds.)

(When we put , we get the same equation.)

(a) If , then , but . The solution is rejected.

(b) If

Using quadratic equation formula,

But since we have

,

, where is a free parameter.

**Complete solution:**

, where are parameters.

**Yue Kwok Choy**

**11-2-2016**